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UNITED STATES NAVAL POSTGRADUATE SCHOOL

DEPARTMENT OF AERONAUTICS

TECHNICAL NOTE

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RANGE OF ORIFICE MEASUREMENTS
FOR MAPPING OF ALLIS-CHALMERS
AXIAL-FLOW COMPRESSOR

by

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PREPARED BY: ____________________ __________

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1. SUMMARY:

A compressor map (Fig. 1) of the VA-312 compressor has been established with approximate methods. Figs. 4 and 5 show this map with lines of constant pressure differences h_w for the 7.9 and the 9 in. orifices which are available to determine the flow rate of the compressor. Fig. 4 shows the conditions if the after-cooler reduces the air temperature to 100°F ahead of the orifice, whereas Fig. 5 shows the conditions without after-cooling. For temperatures of 100°F the 9 in. orifice plate can be used to cover most of the operating range of the compressor. The pressure differences across it can be measured with the water-filled Meriam Vernier Manometers of 40 in. measuring height. For more accurate measurements at compressor speeds of abt. 7000 rpm and lower, it is recommended to install the 7.9 in. orifice plate. For operations at 12,00 rpm without after-cooling the 9 in. orifice plate requires a differential manometer with a water column of at least 50 in. Below about 10,500 rpm the 40 in. Vernier Manometer can be used, but the 7.9 in. orifice plate should be installed only for measurements at speeds of and below 6000 rpm.

2. SYMBOLS:

C	Orifice Coefficient (Eq. 12, Ref. 2)
G	Factor (Eq. 8)
HP	Horsepower absorbed by Compressor
ΔH_{is}	Isentropic Enthalpy Rise in Compressor (Btu/lb)
N	Speed (rpm)
P_1	Pressure ahead of orifice (in. Hg)
R	Gas Constant (ft/^{o}R)
T_o	Ambient Temperature (^{o}R)
T_1	Temperature ahead of orifice (^{o}R)
ΔT	Temperature Rise in Compressor (^{o}R)
U	Peripheral Speed of Compressor (ft/sec)
$(VF)_o$	Suction Volume Flow Rate (cfm)
X	see Eq. 3
Y	see Eq. 2
Y_1	see Ref. 2
c_p	Specific Heat at Constant Pressure (Btu/(lb,^{o}F))
h_w	Pressure Difference across Orifice (in. H_2O)
Δh_{is}	Isentropic Enthalpy Rise per Stage (Btu/lb)
p_o	Ambient Pressure (psia)
p_1	Compressor Discharge Pressure (psia)
v_o	Specific Volume of ambient air (ft^3/lb)
w	Flow Rate (lb/sec)
α	see Ref. 2
$\gamma = c_p/c_v$	
ζ	see Ref. 2
η	Stage Efficiency
η_c	Compressor Efficiency
τ^*	Deflection Coefficient of Stage
φ	Flow Coefficient of Stage

3. INTRODUCTION:

The Allis-Chalmers axial-flow compressor can be operated at speeds between about 4,800 and 12,200 rpm. Since this compressor will be used for tests it is necessary to measure its flow rate for the whole range of operating conditions. Fig. 1 of Ref. 2 shows the arrangement of the flow orifice in the discharge pipe of the compressor. Two interchangeable orifice plates with 7.9 and 9 in. orifice diameter are available. The purpose of this note is to establish whether these orifices make it possible to measure the flow rates at all operating conditions with the available manometers. The pressure differences across the orifices will be measured by the 30 in. or the 40 in. Meriam Vernier Manometers which are installed in the Control Room of the Compressor Laboratory. The pressure ahead of the orifice can be measured either by a manometer board (measuring height about 90 in.) or by precision gages.

4. COMPRESSOR MAP:

Ref. 1 gives the expected compressor performance at different speeds for inlet conditions of 14.7 psia and 80°F. Since the given data look doubtful at lower speeds it was decided to establish an approximate compressor map by using the performance of Ref. 1 at 11,250 rpm. These data are given in Table I.

Table I. PERFORMANCE AT 11,250 RPM (Ref. 1)

Point	1	2	3	4	5
Inlet Volume Flow $(VF)_o$ cfm	8900	9500	10,000	10,500	10,700
Pressure Ratio p_1/p_o	2.95	2.83	2.62	2.17	1.7
Horse Power HP (at coupling)	975	980	890	700	550
η_c (Eq. 10) %	74.3	75.4	80.0	83.4	71.5

Point 1 of Table I is on the surge line of the compressor, and the maximum flow rate is only slightly higher than the flow rate at point 5.

In Ref. 3, p.386, it is shown that the isentropic enthalpy rise Δh_{is} in a stage of an axial-flow compressor is

$$\Delta h_{is} = \eta \tau^* \frac{U^2}{gJ} \tag{1}$$

where τ^* is the dimensionless deflection coefficient, η the stage efficiency, and U the peripheral speed. If ΔH_{is} is the isentropic enthalpy rise of all stages there is, since U is proportional to the speed N,

$$\Delta H_{is} = \Sigma (\eta \tau^*) \frac{U^2}{gJ} = Y N^2 \tag{2}$$

The volume flow rate $(VF)_o$ is proportional to the through-flow velocity V_a which is related to U by $V_a = \varphi U$ (Eq. 13 (16), p. 338, Ref. 3). Hence

$$(VF)_o = (\text{constant}) \varphi U = X N \tag{3}$$

For a single stage the parameters τ^*, η, and φ are related to each other (see Fig. 13 (9), p. 352, Ref. 3). It is now assumed that the coefficient Y of Eq. 2 for the whole compressor is directly related to the coefficient X of Eq. 3 independent of the speed N.

In actuality the quantity Y is a function of N also because of the change of the specific volume of the fluid. At higher speeds, where high pressure ratios are produced, the specific volume at the compressor discharge is considerably lower than that at the compressor inlet, whereas at low speeds and low pressure ratios these specific volumes are more equal. If the compressor at the design point is laid out, say, for a constant through-flow velocity V_a, and constant values of φ for all stages, there is obtained a particular shape of the meridional flow channel, with the blade heights gradually decreasing from the first to the last stage

because of the decreasing specific volume of the fluid. If the same compressor runs at lower speeds the change in specific volume from the first to the last stage is smaller because of the lower pressure rise in the machine. Hence, the through-flow velocity V_a will increase from the first to the last stage, and the stages will not operate at the same flow coefficient φ. Thus, even if the first stage at the lower speed has a value of φ equal to that at the design point (φ_d), the flow coefficient φ of the last stage will be higher than φ_d. The pressure rise (or $\eta\ \tau^*$) of the last stage is lower than that of the first stage, and its efficiency will be lower also. Referring to Fig. 13 (9), p.353 of Ref. 3, it is possible even that the last stages have values of φ that are larger than those for which the product $\eta\ \tau^*$ becomes zero. These stages then work as turbine stages and the pressure at the compressor discharge may be lower than the pressure that occurs at some intermediate stage of the machine. This discussion shows that the establishing of a compressor map is not a simple matter. It is necessary to apply the equation of continuity for the determination of the flow coefficients φ of the different stages, which then must be used to establish τ^* and η to arrive at the overall performance.

If the influence of N on Y of Eq. 2 is ignored the performance map thus established will be of an approximate nature, and the deviations from the actual operating characteristics become greater the more the speeds differ from 11,250 rpm. Eq. 2, written for a perfect gas with $\gamma = c_p/c_v$ = constant

$$\Delta H_{is} = c_p\ T_o \left[\left(\frac{P_1}{P_o} \right)^{(\gamma-1)/\gamma} - 1 \right] = Y\ N^2$$

or,

$$Y = c_p \frac{\left[\left(\frac{P_1}{P_o} \right)^{(\gamma-1)/\gamma} - 1 \right]}{(N/\sqrt{T_o})^2} \qquad (4)$$

and

$$\frac{p_1}{p_o} = \left(1 + \frac{Y\,(N/\sqrt{T_o})^2}{c_p}\right)^{\frac{\gamma}{\gamma-1}} \tag{5}$$

The weight flow rate w (lb/sec) is from Eq. 3

$$w = \frac{(VF)_o}{v_o} = \frac{X\,N\,p_o}{R\,T_o} = \frac{X\,(N/\sqrt{T_o})\,p_o}{R\sqrt{T_o}}$$

Thus

$$X = \frac{w\sqrt{T_o}}{p_o}\,\frac{R}{N/\sqrt{T_o}} \tag{6}$$

and

$$\frac{w\sqrt{T_o}}{p_o} = \frac{X\,(N/\sqrt{T_o})}{R} \tag{7}$$

These relations show that the behavior of a compressor that delivers a particular gas (constant values of c_p, γ, and R) is best expressed in terms of the referred flow rate $w\sqrt{T_o}/p_o$, the referred speed $N/\sqrt{T_o}$, and the pressure ratio p_o/p_o.

Eqs. 4 and 6 are now used to determine the values of Y and X, respectively, from the data of Table I. For these operating conditions, there are $T_o = 540^oR$, $p_o = 14.7$ psia, and $N/\sqrt{T_o} = 11{,}250/\sqrt{540} = 484$ rpm$/\sqrt{^oR}$. For $\gamma = 1.4$, $c_p = .24$, and $R = 53.345$ the values of X and Y are listed in Table II.

Table II. Values of X and Y of Eqs. 6 and 4 from Data of Table I.

Point	1	2	3	4	5
X	1.901	2.030	2.135	2.230	2.283
$Y \times 10^6$	.3708	.3543	.3242	.2536	.1676

The name plate of the Allis-Chalmers Compressor gives the design point of the machine as follows:

$(VF)_o$ = 11,200 cfm at 70°F, 14.5 psia

N = 12,200 rpm

$P_1/P_o = 3$

Thus,

$$\frac{w\sqrt{T_o}}{P_o} = 21.9 \ \frac{\text{lb}}{\text{sec}} \ \frac{\sqrt{{}^\circ R}}{\text{psia}}$$

$$N/\sqrt{T_o} = 530 \text{ rpm}/\sqrt{{}^\circ R}$$

The values of Table II are used to determine the pressure ratios and the referred flow rates by means of Eqs. 5 and 7, respectively, for referred speeds of 100% to 40% of 530 rpm/$\sqrt{{}^\circ R}$. In accordance with Ref. 4 the hydraulic speed change coupling of the installation must make it possible to operate the compressor in this range of speeds.

The compressor map obtained in this manner is shown in Fig. 1. It can be seen that the referred flow rate in accordance with the name plate data is by about 2½% lower than the calculated one, but about 8% higher than the minimum performance specified in Ref. 4 (10,000 cfm at 70°F, 14.5 psia).

5. DIFFERENTIAL PRESSURES ACROSS FLOW ORIFICES:

The flow rate of the compressor can be determined with the orifice meter shown in Fig. 1 of Ref. 2. Eq. 12 of Ref. 2 gives the relation to calculate the flow rate. In this equation the quantities α, Y_1, and ζ do not differ greatly from unity. Then, approximately,

$$h_w = \left(\frac{w\sqrt{T_1}}{\sqrt{P_1}}\right)^2 \frac{1}{C^2}$$

However, since P_1 is the pressure ahead of the orifice in inches of mercury, there is, with p_1 in psia,

$$h_w = \left(\frac{w\sqrt{T_1}}{\sqrt{p_1}}\right)^2 \frac{1}{(2.0367)\ C^2}$$

or

$$h_w = \left(\frac{w\sqrt{T_1}}{p_1}\right)^2 \frac{p_1}{2.0367\ C^2}$$

and

$$h_w = \left(\frac{w\sqrt{T_o}}{p_o}\right)^2 \frac{T_1/T_o}{p_1/p_o} \frac{p_o}{2.0367} \frac{1}{C^2}$$

$$= \left(\frac{w\sqrt{T_o}}{p_o}\right)^2 \frac{T_1/T_o}{p_1/p_o}\ G \qquad (8)$$

This relation contains the referred flow rate of Eq. 7, and p_1/p_o which is equal to the pressure ratio of the compressor if the losses in the after-cooler are ignored. The local ambient pressure is about 14.7 psia. Although the value of C depends on the type of pressure taps used, it is assumed that C has average values of 6.7 and 4.78 for the 9 in. and the 7.9 in. orifice diameters, respectively (see Tables I and II of Ref. 2). Hence the factor G of Eq. 8 equals 0.161 and 0.317, respectively, for the 9 in. and the 7.9 in orifices.

For operations with the after-cooler the air temperature T_1 ahead of the orifice is about 560°R. The outside temperature is taken as 70°F. If the air is not cooled the rise ΔT of the air temperature in the compressor is

$$\Delta T = T_1 - T_o = \frac{T_o}{\eta_c}\left[\left(\frac{p_1}{p_o}\right)^{(\gamma-1)/\gamma} - 1\right]$$

or, with Eq. 4,

$$\Delta T = \frac{T_o}{\eta_c} \frac{Y(N/\sqrt{T_o})^2}{c_p} \tag{9}$$

The quantity η_c in Eq. 9 is the compressor efficiency. There is, with Eq. 2,

$$\eta_c = \frac{w \ \Delta H_{is} \ (778)}{HP \ (550)} = \frac{(VF)_o \ Y \ N^2 \times 778}{v_o \ (60) \ (550) \ HP}$$

For N = 11,250 rpm, v_o = 13.6 ft^3/lb,

$$\eta_c = (0.2195) \ \frac{(VF)_o \ Y \times 10^6}{HP} \tag{10}$$

Table I shows the efficiencies η_c for the different flow rates. The high value obtained for point 4 must be error, since the efficiency at this flow rate is more likely equal to about 78%. These efficiencies were used in Eq. 9 to establish the ratios T_1/T_o for operation without after-cooling.

The results obtained from Eq.8 are shown in Figs. 2 and 3 for different referred speeds, and the two orifice diameters. Figs. 3 and 4 represent the estimated compressor map of Fig. 1 with lines of constant differential pressures h_w across the orifice, for operations with and without after-cooling, respectively.

Fig. 4 demonstrates that the 9 in. orifice and the 40 in. manometer can be used to measure the flow rate over most of the operating conditions of the compressor if the compressed air is cooled. To increase the measuring accuracy at speeds of abt. 60% and lower, the 7.9 in. orifice should be installed.

If the air is not cooled after the compressor the 9 in. orifice produces differential pressures h_w that are higher than 40 in. of water at 100% speed and small pressure ratios, but it is possible

to use the 7.9 in. orifice with the 40 in. manometer at speeds of 50% and lower. At full speed it is necessary to use a differential pressure manometer with at least 50 in. measuring height if the pressure ratio of the compressor is lower than about 2.8.

6. REFERENCES:

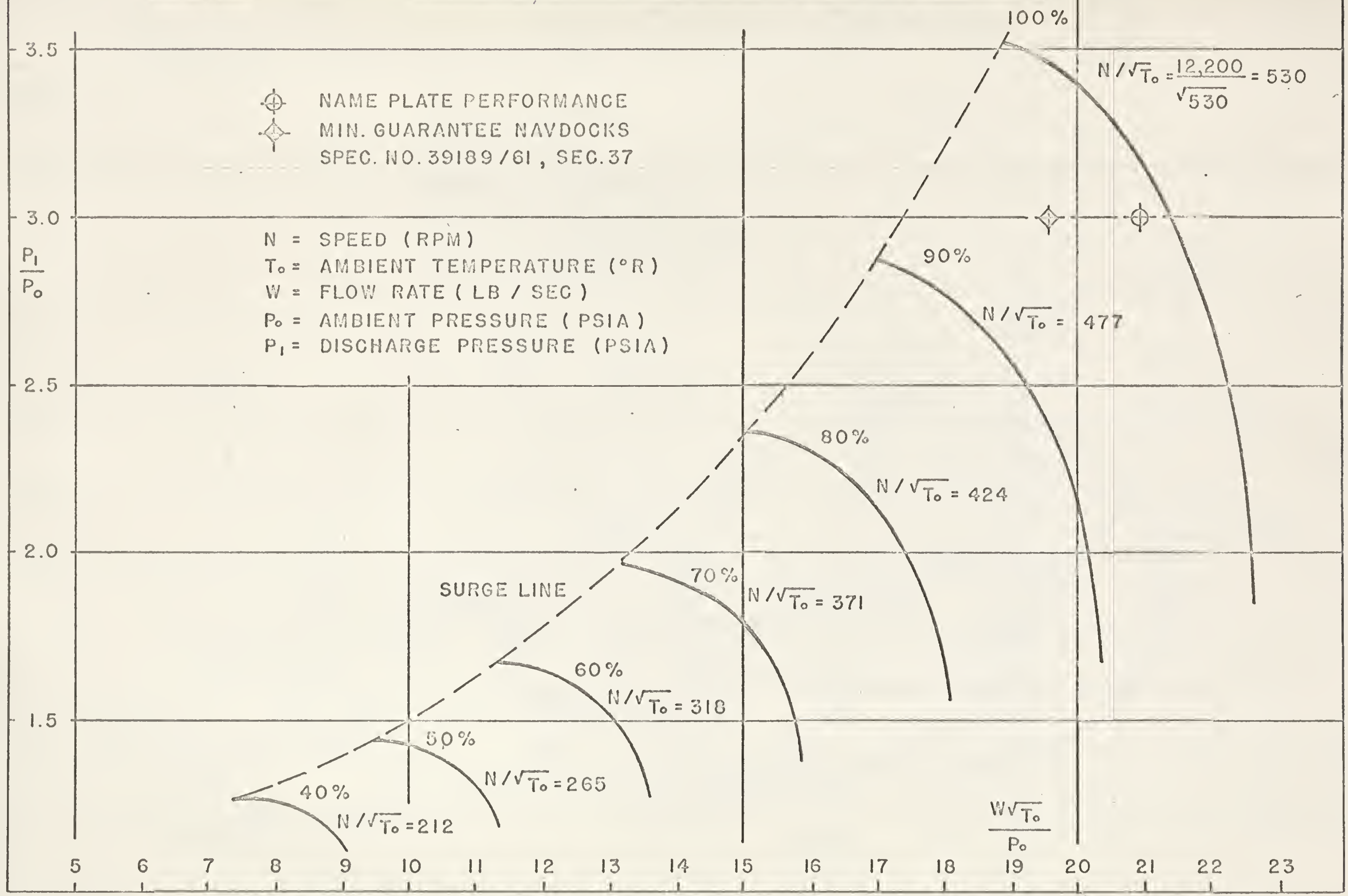
1. CURVE BL 6126, ALLIS-CHALMERS MFG. CO. "Expected Performance of VA-312 Axial Compressor", Aug. 4, 1961.

2. TECHNICAL NOTE, TN No. 63T-2, Aeronautics Department, USNPGS, August 1963.

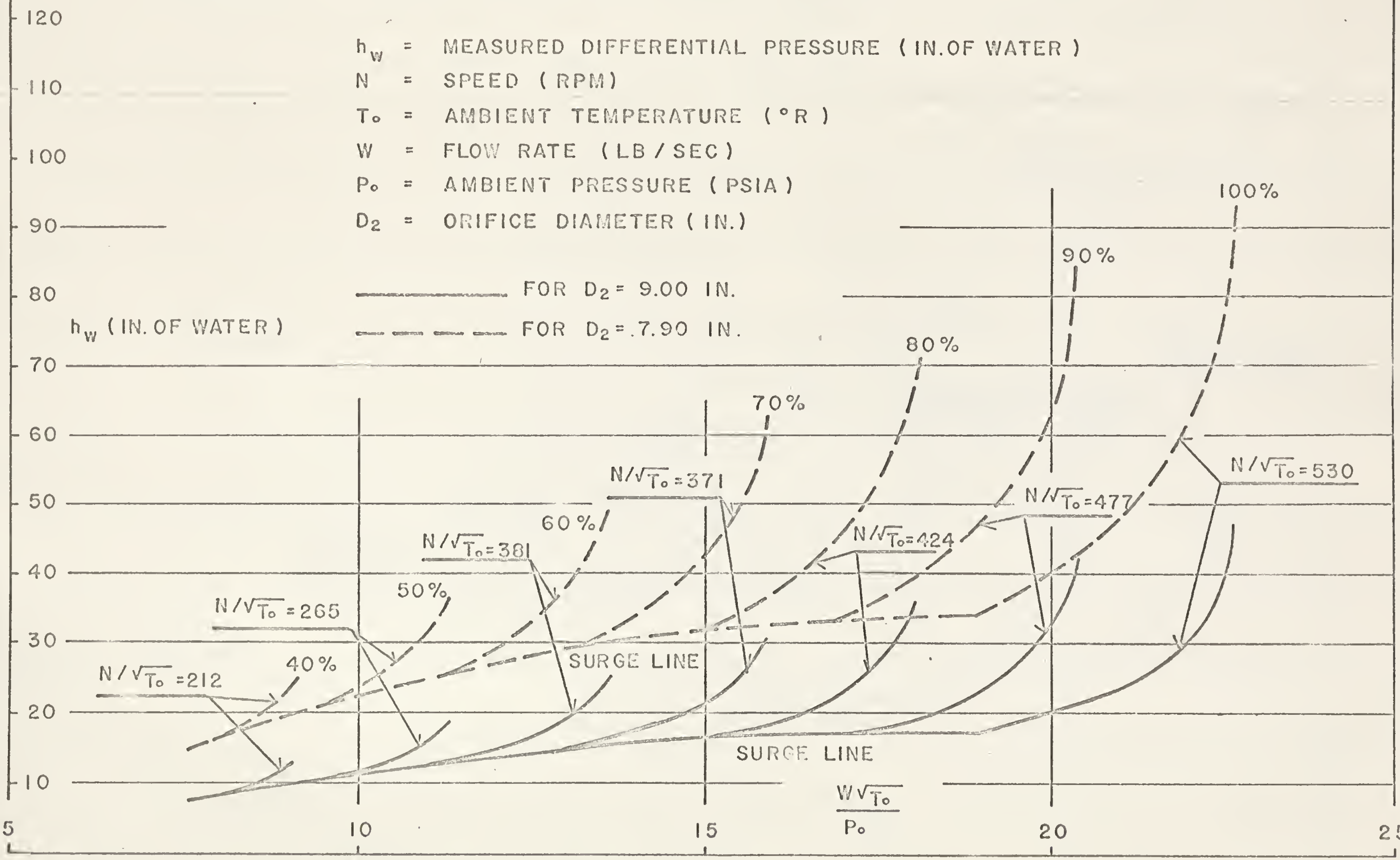
3. Vavra, M. H., AEROTHERMODYNAMICS AND FLOW IN TURBO-MACHINES, Wiley and Sons, New York, 1960.

4. NAVDOCKS SPEC. No. 39189/61 for Astro/Aero Propulsion Laboratories, USNPGS, Sec. 37.

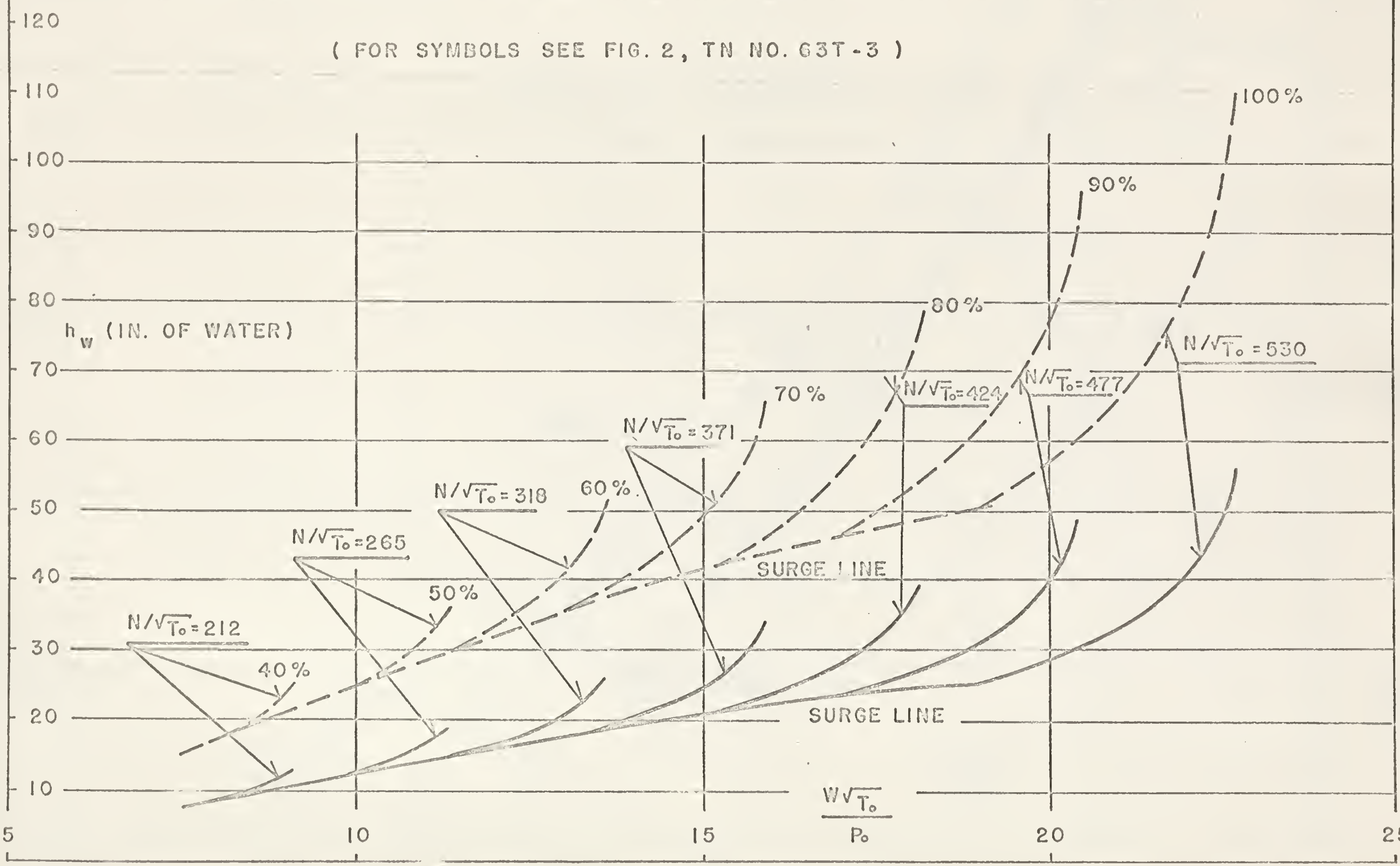
VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG. 1
	ESTIMATED MAP OF ALLIS-CHALMERS AXIAL—	TECH. NOTE
AUGUST 63	FLOW COMPRESSOR	NO. 63T-3



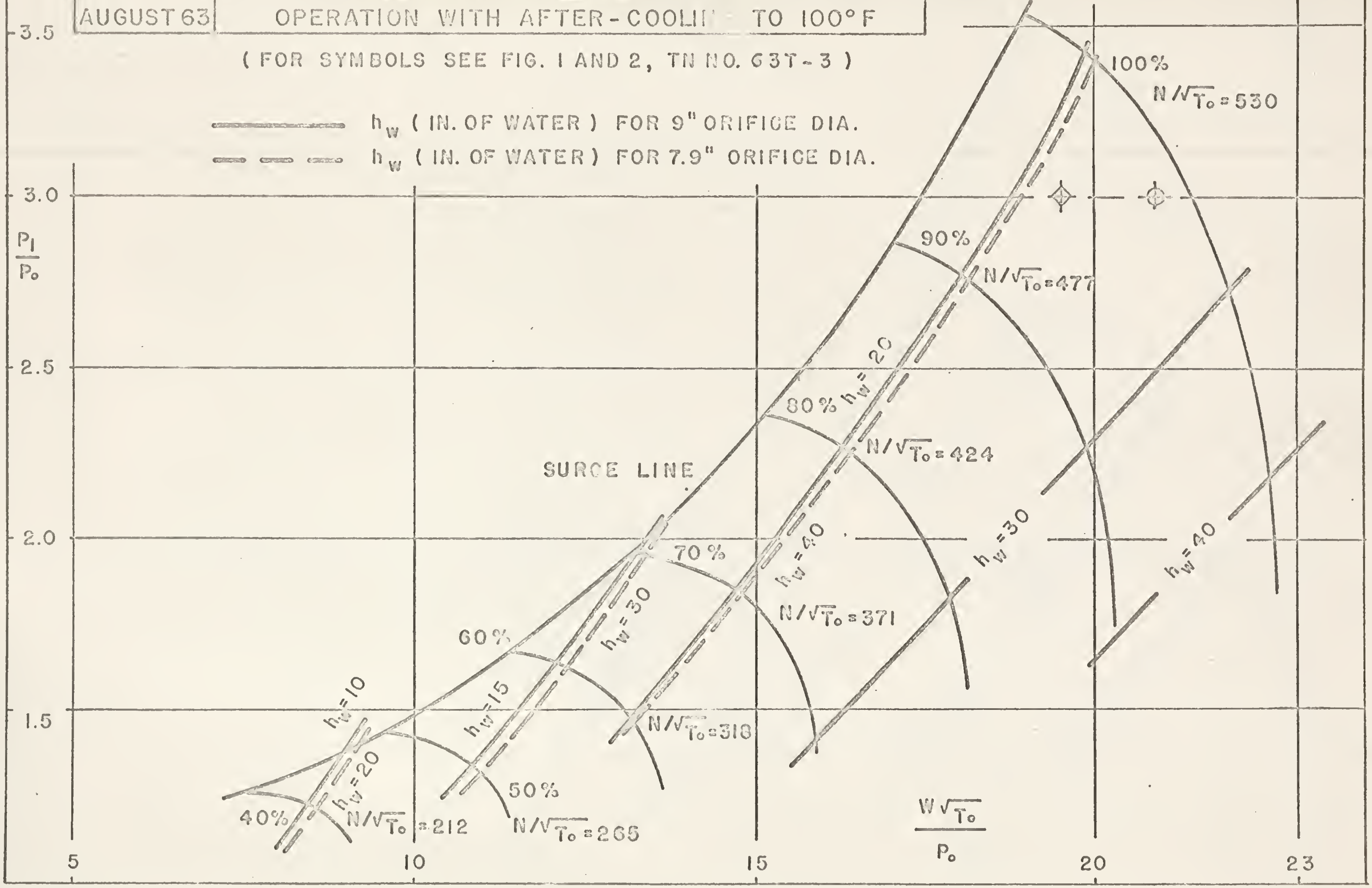
VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG. 2
	DIFFERENTIAL PRESSURE h_w ACROSS AIR FLOW ORIFICES INSTALLED IN 12.IN.DIA. DISCHARGE PIPE OF ALLIS-CHALMERS AXIAL-FLOW COMPRESSOR	TECH. NOTE TN NO. 63T - 3
AUGUST63	OPERATION WITH AFTER-COOLING TO 100°F.	



VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG. 3
	DIFFERENTIAL PRESSURE h_w ACROSS AIR FLOW ORIFICES INSTALLED IN 12 IN. DIA. DISCHARGE PIPE OF ALLIS-CHALMERS AXIAL-FLOW COMPRESSOR	TECH. NOTE TN NO. 63T-3
AUGUST 63	OPERATION WITHOUT AFTER-COOLING	



VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG. 4
	ESTIMATED MAP OF ALLIS-CHALMERS AXIAL-FLOW COMPRESSOR WITH LINES OF CONTANT PRESSURE DIFFERENCES h_w ACROSS ORIFICES.	TECH. NOTE TN NO. 63T-3
AUGUST 63	OPERATION WITH AFTER-COOLING TO 100°F	



VAVRA	USNPGS, DEPT. OF AERONAUTICS, PROPULSION LAB.	FIG 5
	ESTIMATED MAP OF ALLIS-CHALMERS AXIAL-FLOW COMPRESSOR WITH LINES OF CONSTANT PRESSURE DIFFERENCES h_w ACROSS ORIFICES.	TECH. NOTE TN NO. 63T-3
AUGUST 63	OPERATION WITHOUT AFTER-COOLING